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Renormalization of a 2D scalar–tensor gauge theory

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Abstract. A gauge invariant interaction between a symmetric second-rank tensor field and a scalar field is examined in detail in exactly two dimensions. The quantum dynamics of the tensor field are determined solely by the gauge fixing term in the effective action. The divergence structure of this model is analysed. All divergences can be removed by renormalizations of the scalar field wavefunction (both linear and nonlinear), and the non-polynomial interaction functions of the scalar field. We illustrate how operator regularization can be used to compute radiative effects to one-loop order.

1. Introduction

Two-dimensional field theories have attracted much attention over the past decade. In particular, a lot of study has been done of the two-dimensional nonlinear sigma models. The main reason for this interest is the connection between these models and the higher-dimensional string theories. It was Friedan [1] who first showed that these models are renormalizable, but not in the multiplicative (or, more correctly, linear) sense. The sigma models are also endowed with a geometrical structure.

If the intention is to try out new calculational methods in a two-dimensional theory, the geometrical structure can complicate matters. For this reason we wish to examine a two-dimensional field theory which is (nonlinearly) renormalizable but which is structurally simpler than the nonlinear sigma model. It is our hope that this model can be used to learn about the quantum dynamics of other (nonlinearly) renormalizable theories in two dimensions such as, for instance, the (nonlinear) sigma model on a curved two-dimensional background space.

Recently a renormalizable (in two dimensions) gauge theory involving a massless symmetric second-rank tensor field‡ $h_{\mu\nu}$ and a massive scalar field ϕ was proposed for this purpose [2]. In this paper we refine the model and discuss more fully the manner in which it is to be renormalized. The model has two unusual features:

(i) as the Lagrangian of the classical symmetric tensor field vanishes identically in two dimensions, the effective Lagrangian in this sector is given by the gauge fixing term;

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‡ We use the terms 'spin-2' and 'symmetric second-rank tensor' interchangeably, in accordance with [2]. The origin of our useage is in four-dimensional theories; in two dimensions the concept of spin is not properly defined.

(ii) as all the fields occurring in the model are dimensionless, nonlinear renormalizations are allowed and, indeed, necessary, as we shall see in section 3.

Clearly, the choice of gauge fixing term in the Lagrangian is of some importance. We must select a gauge fixing term which yields an effective Lagrangian which can then be used in the proposed calculational framework. It is important to use this criterion in choosing the gauge fixing term—indeed, if two-loop calculations are attempted using the gauge fixing term of [2] then intractable divergences of the form

$$\lim_{c\to 0}\int_0^1 \mathrm{d}x\, x^{-c-2}(1-x)^c$$

are encountered. A choice of gauge fixing term more appropriate to the calculational method used overcomes this apparent difficulty.

The next step is to analyse the divergence structure of the unregularized theory, and to provide a heuristic indication of how the divergences can be removed by renormalization. In four dimensions it is straightforward to provide a renormalization scheme symmetry indicating just how all of these infinities are to be removed. The non-zero dimensions of the fields in four dimensions have the effect of severely restricting both the possible terms which can occur in the Lagrangian and the possible types of renormalizations. In two dimensions the fields are dimensionless and the restrictions are fewer.

The logarithmic infinities appearing in the Green function with just external scalars, $\langle\phi^{2n}\rangle$, are dealt with by means of both linear and nonlinear renormalizations of the scalar field ϕ. This is possible only because ϕ is dimensionless. Gauge invariance ensures that $h_{\mu\nu}$ does not need to be renormalized. Quadratic divergences are removed from Green functions of the form $\langle\phi^{2n}\rangle$ by renormalizing a function $m^2V(\phi)$ occurring in the initial Lagrangian. A similar device is used in the nonlinear sigma model [1, 3]. Finally, the remaining logarithmic divergences appearing in the Green functions $\langle h_{\mu\nu}\phi^{2n}\rangle$ indicate that we must renormalize a function $F(\phi)$ occurring in the interaction term.

The calculation of explicit Green functions in this model requires the specification of both a regularization procedure and a subtraction procedure. It is conventional nowadays to use dimensional regularization (DR) together with some consistent subtraction scheme, such as MS. Such an approach is not suited to this model as it inevitably involves continuing away from two dimensions. But it is only in exactly two dimensions that the (symmetric) tensor field Lagrangian vanishes, and that all the fields are dimensionless. Thus, even though DR respects the gauge invariance of the model, we opt instead for an alternative symmetry-preserving regularization scheme, namely operator regularization (OR) [4] as it leaves unchanged the dimensions of the space on which the model is defined. It has the additional advantage of eliminating the need to specify a separate subtraction scheme. No explicit divergences occur at any stage of the calculations—the renormalizations are carried out implicitly.

This paper is organized as follows. Section 2 contains the details of the model—we introduce the effective Lagrangian. In section 3 we examine the divergence structure of the bare theory and indicate how renormalizations of mass, wavefunctions (both linear and nonlinear renormalizations) and the non-polynomial (interaction) functions of the scalar field can serve to eliminate all the divergences. In section 4, operator regularization is applied to the model and some sample calculations are carried out to illustrate the unusual renormalization features highlighted in section 3. Some concluding remarks are included in section 5, while an appendix contains the necessary details to carry out the calculations reported in the text.

2. The model

We begin with the scalar field Lagrangian

$$\mathscr{L}_A = -\tfrac{1}{2}(\phi p^2 \phi + m^2 V(\phi)) \qquad (p \equiv -\mathrm{i}\partial) \tag{1a}$$

where $V(\phi)$ is a non-polynomial function of the scalar field ϕ satisfying $V(\phi) = V(-\phi)$, $V(0) = 0$ and $V''(0) = 2$. This field ϕ is to be coupled to a spin two field $h_{\mu\nu}$ whose Lagrangian [2] is

$$\mathscr{L}_B = \tfrac{1}{2}(\partial_\mu h_{\lambda\lambda} - 2\partial_\lambda h_{\lambda\mu})^2 - (\partial_\lambda h_{\mu\nu})^2 + \tfrac{1}{2}(\partial_\mu h_{\lambda\lambda})^2. \tag{1b}$$

In two dimensions, $\mathscr{L}_B$ vanishes identically. This Lagrangian is invariant under the gauge transformation

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu \qquad \phi \to \phi. \tag{2}$$

This gauge invariance is preserved if we couple $h_{\mu\nu}$ to ϕ with the coupling

$$\mathscr{L}_I = \tfrac{1}{2} h_{\mu\nu}(\partial_\mu \partial_\nu - \delta_{\mu\nu}\partial^2) F(\phi) \tag{3}$$

where $F(\phi)$ is a (possibly non-polynomial) function of the scalar field ϕ satisfying $F(\phi) = F(-\phi)$ and $F(0) = 0$.

The full gauge invariant classical Lagrangian is

$$\mathscr{L}_{cl} = -\tfrac{1}{2}(\phi p^2 \phi + m^2 V(\phi)) + \tfrac{1}{2}(\partial_\mu h_{\lambda\lambda} - 2\partial_\lambda h_{\lambda\mu})^2 - (\partial_\lambda h_{\mu\nu})^2 + \tfrac{1}{2}(\partial_\mu h_{\lambda\lambda})^2 + \tfrac{1}{2} h_{\mu\nu}(\partial_\mu \partial_\nu - \delta_{\mu\nu}\partial^2) F(\phi). \tag{4}$$

This Lagrangian is more general than that considered previously in [2]. The earlier model corresponds to a particular choice of the functions $V(\phi)$ and $F(\phi)$, namely $V(\phi) = \phi^2$ and $F(\phi) = \lambda\phi^2$. However, as ϕ is dimensionless in two dimensions it is possible to include the more general form (without necessarily prejudicing the renormalizability of the model, as we shall see in section 3). The occurrence of non-polynomial functions of the scalar field in the classical Lagrangian is a feature of not only the model proposed in this paper, but also of the nonlinear sigma models. In the sigma models [1, 3] it is the metric $g_{ij}(\phi)$ and the function $V(\phi)$ which are non-polynomial in ϕ, while in this model it is the functions $F(\phi)$ and $V(\phi)$ that are non-polynomial. It must be emphasized that it is only in two dimensions that the occurrence of such non-polynomial interactions does not give rise to additional dimensional coupling constants, as scalar fields are dimensionless only in two dimensions.

In order to break the gauge invariance of $\mathscr{L}$ under the gauge transformations of (2), we choose a gauge fixing term of the form

$$\mathscr{L}_{gf} = -(A\partial_\mu h_{\mu\nu} + B\partial_\nu h_{\mu\mu})^2. \tag{5}$$

We are interested in identifying the term in the effective Lagrangian $\mathscr{L}_{cl} + \mathscr{L}_{gf}$ which is bilinear in $h_{\mu\nu}$. The most useful way to do this is to avail of the known set of projection operators [5] for the symmetric second-rank tensor field. These projection operators are defined, in n dimensions, by

$$(P^{(2)})_{\alpha\beta,\gamma\delta} = \tfrac{1}{2}(\theta_{\alpha\gamma}\theta_{\beta\delta} + \theta_{\alpha\delta}\theta_{\beta\gamma}) - \frac{1}{n-1}\theta_{\alpha\beta}\theta_{\gamma\delta} \tag{6a}$$

$$(P_m^{(1)})_{\alpha\beta,\gamma\delta} = \tfrac{1}{2}(\theta_{\alpha\gamma}\omega_{\beta\delta} + \theta_{\beta\gamma}\omega_{\alpha\delta} + \theta_{\alpha\delta}\omega_{\beta\gamma} + \theta_{\beta\delta}\omega_{\alpha\gamma}) \tag{6b}$$

$$(P_s^{(0)})_{\alpha\beta,\gamma\delta} = \frac{1}{n-1}\,\theta_{\alpha\beta}\theta_{\gamma\sigma} \tag{6c}$$

$$(P_\omega^{(0)})_{\alpha\beta,\gamma\delta} = \omega_{\alpha\beta}\omega_{\gamma\delta} \tag{6d}$$

$$(P_{s\omega}^{(0)})_{\alpha\beta,\gamma\delta} = \frac{1}{\sqrt{n-1}}\,\theta_{\alpha\beta}\omega_{\gamma\delta} \tag{6e}$$

$$(P_{\omega s}^{(0)})_{\alpha\beta,\gamma\delta} = \frac{1}{\sqrt{n-1}}\,\omega_{\alpha\beta}\theta_{\gamma\delta} \tag{6f}$$

in terms of the quantities

$$\theta_{\alpha\beta} = \delta_{\alpha\beta} - \frac{p_\alpha p_\beta}{p^2} \tag{7a}$$

$$\omega_{\alpha\beta} = \frac{p_\alpha p_\beta}{p^2}. \tag{7b}$$

The upper labels on the projectors P refer to the spin associated with the projection operator in the case $n=4$. The first four projectors taken together $\{P^{(2)}, P_m^{(1)}, P_\omega^{(0)}, P_s^{(0)}\}$ from a complete set of orthogonal projection operators. The remaining two projection operators are not orthogonal to the elements of this set. They are in fact transition operators between the two spin-0 subspaces labelled by ω and s.

When we restrict our attention to two dimensions, the set of projection operators simplifies, as $P^{(2)}=0$ identically. This allows us to rewrite $\mathscr{L}_{\rm gf}$ using (5)-(7), as

$$\mathscr{L}_{\rm gf} = -\tfrac{1}{4}h_{\alpha\beta}[\tfrac{1}{2}A^2P_m^{(1)} + (A+B)^2P_\omega^{(0)} + B^2P_s^{(0)} + (A+B)B(P_{s\omega}^{(0)} + P_{\omega s}^{(0)})]_{\alpha\beta,\gamma\delta}h_{\gamma\delta}. \tag{8}$$

For arbitrary A and B, $\mathscr{L}_{\rm gf}$ depends on the transition operators $P_{s\omega}^{(0)}$ and $P_{\omega s}^{(0)}$. However, if $A+B$ vanishes we see that these operators do not contribute and $\mathscr{L}_{\rm gf}$ can be written solely in terms of the orthogonal projection operators. For convenience we choose $A=-B=2$, to find

$$\mathscr{L}_{\rm gf} = -\tfrac{1}{2}h_{\alpha\beta}p^2(P_m^{(1)} + 2P_s^{(0)})_{\alpha\beta,\gamma\delta}h_{\gamma\delta}. \tag{9}$$

This particular combination of orthogonal projection operators will turn out to be particularly useful. (We note in passing that in [2] the choice for $\mathscr{L}_{\rm gf}$ corresponded to $A=0$, $B=\tfrac{1}{2}$.)

3. Divergence structure of the bare model

Before proceeding to demonstrate how particular Green functions are to be computed in this model, we wish to show that the model is, indeed, renormalizable. To do this it is first necessary to consider the divergence structure of the bare model.

In the conventional Feynman approach vertices are quadratic in moments and propagators are quadratic in inverse momenta. This would suggest that all unregulated Green functions are naively quadratically divergent. However, as external spin-2 fields are transverse (due to the gauge invariance of (4)) the degree of divergence for any Green function is reduced by two for each an external field $h_{\mu\nu}$. The role of gauge invariance here is similar to the role of gauge invariance in QED.

Thus, power counting and gauge invariance gives rise to the following divergence structure.

(i) All Green functions $\langle \phi^{2n} \rangle$ have logarithmically and quadratically divergent parts.

(ii) All Green functions $\langle h\phi^{2n} \rangle$ are logarithmically divergent.

(iii) All Green functions $\langle h^m \phi^{2n} \rangle$, $m \geqslant 2$, $n \geqslant 0$, are finite.

This divergence structure cannot be treated by means of a standard linear renormalization scheme. However, it is posible to renormalize the theory provided we introduce both nonlinear field renormalizations and renormalizations of the non-polynomial interaction functions. One renormalization scheme which will deal with the rich divergence structure of the model is as follows.

(a) The logarithmic divergence in the Green function $\langle \phi^2 \rangle$ is eliminated by means of a linear wavefunction renormalization $Z\phi$.

(b) The further logarithmic divergences in the Green function $\langle \phi^{2n} \rangle$, for $n > 1$, can be eliminated by nonlinear wavefunction renormalization of the form $Z_n \phi^n$.

The relationship between the bare and renormalized resulting from the wavefunction renormalizations of (a) and (b) is

$$\phi_0 = \sum_{n \geqslant 1} Z_n \phi^n. \tag{10}$$

(c) There is no wavefunction renormalization for the field $h_{\mu\nu}$ as all Green functions of the form $\langle h^m \phi^{2n} \rangle$ for $m \geqslant 2$, $n \geqslant 0$ are finite.

(d) The quadratic divergences in Green functions of the form $\langle \phi^{2n} \rangle$ for $n \geqslant 1$ are dealt with by renormalizing the function $V(\phi)$ which occurs in the scalar field Lagrangian of (1). Such a renormalization can be simply viewed as a generalization of the usual mass renormalization. We can see this more clearly if we expand the term in the spin-0 Lagrangian of (1):

$$\begin{aligned} m^2 V(\phi) &= m^1 \left(\frac{1}{2!} V''(0)\phi^2 + \frac{1}{4!} V^{(\mathrm{IV})}(0)\phi^4 + \ldots \right) \\ &= m_2^2 \phi^2 + m_4^2 \phi^4 + \ldots . \end{aligned} \tag{11}$$

This indicates that renormalizing $V(\phi)$ means renormalizing all the expansion coefficients of $V(\phi)$—or just simply renormalizing an infinite set of coupling constants. A device such as this has already been used in the nonlinear sigma model [1, 3].

(e) Finally, the logarithmic divergences divergences occurring in Green functions $\langle h\phi^{2n} \rangle$ for $n > 1$ are eliminated by a renormalization similar to that at stage (d). It is the function $F(\phi)$ which occurs in the interaction Lagrangian of (3) that is to be renormalized. This can be viewed as a generalization of the usual coupling constant renormalization, especially if we expand $F(\phi)$ in powers of ϕ:

$$\begin{aligned} F(\phi) &= \frac{1}{2!} F''(0)\phi^2 + \frac{1}{4!} F^{(\mathrm{IV})}(0)\phi^4 + \ldots \\ &= \lambda_2 \phi^2 + \lambda_4 \phi^2 + \ldots . \end{aligned} \tag{12}$$

Renormalizing each of the expansion coefficients is just renormalizing an infinite set of coupling constants. While in this model, and in the sigma models, it appears, at a perturbative level, as if we are renormalizing infinite sets of coupling constants, nevertheless these renormalizations can be interpreted more simply as renormalizations of the non-polynomial functions—in this case, $V(\phi)$ and $F(\phi)$—directly.

In this section we have been discussing the unregulated theory. Some symmetry-preserving regulating procedure must be invoked to render meaningful the renormalization procedure outlined above. In the following section we employ operator regularization. When calculating regulated Green functions we make use of Schwinger expansions rather than the Feynman perturbation expansion. No divergences ever occur in the evaluation of these operator-regulated Green functions. Nevertheless, when the regulating procedure is turned off the divergence structure is identical in the Schwinger and Feynman approaches.

4. Sample computations in the operator-regulated model

In the preceding section we discussed the divergence structure of the bare theory and argued that the theory is renormalizable by means of both nonlinear wavefunction renormalizations and the renormalization of the non-polynomial interaction terms. In this section we examine the regulated theory.

Once a theory is regulated it is possible to compute radiative effects. Operator regularization is particularly well suited to this model for several reasons. It preserves the gauge symmetries of the theory. The regulated theory resides in exactly two spacetime dimensions so that the important features of the classical two-dimensional field theory are retained, namely the vanishing of the free symmetric tensor Lagrangian of ($1b$) and the zero dimensions of all the fields of the theory.

Operator regularization requires the use of background field quantization. This involves first expanding the fields of the theory, h and ϕ, into the sum of classical fields (k and f) and quantum fields (γ and ψ)

$$h_{\mu\nu} = k_{\mu\nu} + \gamma_{\mu\nu} \tag{13a}$$

$$\phi = f + \psi. \tag{13b}$$

Computation of one-loop effects using background fields requires only those terms in the effective Lagrangian $\mathscr{L}_{\rm cl} + \mathscr{L}_{\rm gf}$ bilinear in quantum fields, namely

$$\mathscr{L}^{(2)} = \mathscr{L}^{(2)}_{\rm A} + \mathscr{L}^{(2)}_{\rm B} + \mathscr{L}^{(2)}_{\rm I} + \mathscr{L}^{(2)}_{\rm gf}$$

$$= \tfrac{1}{2}[\gamma_{\mu\nu}, \psi]\begin{bmatrix} p^2(P^{(1)}_m + 2P^{(0)}_s)_{\mu\nu,\alpha\beta} & \tfrac{1}{2}(p_\mu p_\nu - p^2\delta_{\mu\nu})F'(f) \\ \tfrac{1}{2}F'(f)(p_\alpha p_\beta - p^2\delta_{\alpha\beta}) & p^2 + m^2 V''(f) + \tfrac{1}{2}\mathbb{K}F''(f) \end{bmatrix}\begin{bmatrix} \gamma_{\alpha\beta} \\ \psi \end{bmatrix} \tag{14}$$

where

$$\mathbb{K} \equiv k_{\alpha\alpha,\beta\beta} - k_{\alpha\beta,\alpha\beta} \tag{15}$$

is linear in the background field k while $F'(f)$, $F''(f)$ and $V''(f)$ are in general non-polynomial functions of the background field f. The unregulated one-loop generating functional is evaluated to be

$$\Gamma^{(1)} = -\tfrac{1}{2}\ln\det(\mathbf{M}/\mu^2) \tag{16}$$

where the matrix $\mathbf{M}$ (with the dimensions of (mass)^2) is given by

$$\mathbf{M} = \begin{bmatrix} p^2(P^{(1)}_m + 2P^{(0)}_s)_{\mu\nu,\alpha\beta} & \tfrac{1}{2}(p_\mu p_\nu - p^2\delta_{\mu\nu})F'(f) \\ \tfrac{1}{2}F'(f)(p_\alpha p_\beta - p^2\delta_{\alpha\beta}) & p^2 + \tfrac{1}{2}m^2 V''(f) + \tfrac{1}{2}\mathbb{K}F''(f) \end{bmatrix}. \tag{17}$$

The one-loop generating functional $\Gamma^{(1)}$ is unique to an additive background-field-independent factor. We can make use of this ambiguity to rewrite $\Gamma^{(1)}$ in terms of a matrix whose diagonal entries are simpler, for future computations

$$\Gamma^{(1)} = -\tfrac{1}{2}\ln[(\det{}^{-1}\mathbf{X})\det(\mathbf{XM}/\mu^2)] \simeq \tfrac{1}{2}\ln\det(\mathbf{XM}/\mu^2) \tag{18}$$

where

$$\mathbf{X} = \begin{bmatrix} (P_m^{(1)} + \frac{1}{2}P_s^{(0)})_{\mu\nu,\alpha\beta} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{XM} = \begin{bmatrix} p^2(P_m^{(1)} + P_s^{(0)}) & \frac{1}{4}(p_\mu p_\nu - p^2\delta_{\mu\nu})F'(f) \\ \frac{1}{2}F'(f)(p_\alpha p_\beta - p^2\delta_{\alpha\beta}) & p^2 + \frac{1}{2}m^2 V''(f) + \frac{1}{2}\mathbb{K}F''(f) \end{bmatrix}.$$

We now regulate the functional determinant in (18) using operator regularization [4] so that

$$\Gamma^{(1)} = \tfrac{1}{2}\lim_{s\to 0}\frac{\mathrm{d}}{\mathrm{d}s}\zeta(s) \tag{19}$$

where

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty \mathrm{d}t\, t^{s-1}\,\mathrm{Tr}\exp[-t(\mathbf{MX}/\mu^2)]. \tag{20}$$

One-loop Green functions can be obtained from the regulated generating functional $\Gamma^{(1)}$ of (19) by use of the Schwinger expansion [4, 6]

$$\begin{aligned} \mathrm{e}^{\mathbf{A}+\mathbf{B}} = &\int_0^\infty \mathrm{d}\alpha_1\, \mathrm{e}^{\alpha_1\mathbf{A}}\delta(1-\alpha_1) + \int_0^\infty \mathrm{d}\alpha_1\,\mathrm{d}\alpha_2\, \mathrm{e}^{\alpha_1\mathbf{A}}\mathbf{B}\,\mathrm{e}^{\alpha_2\mathbf{A}}\delta(1-\alpha_1-\alpha_2) \\ &+ \int_0^\infty \mathrm{d}\alpha_1\,\mathrm{d}\alpha_3\,\mathrm{d}\alpha_3\, \mathrm{e}^{\alpha_1\mathbf{A}}\mathbf{B}\,\mathrm{e}^{\alpha_2\mathbf{A}}\mathbf{B}\,\mathrm{e}^{\alpha_3\mathbf{A}}\delta(1-\alpha_1-\alpha_2-\alpha_3) \\ &+ \ldots \end{aligned} \tag{21}$$

where A is the part of $\mathbf{MX}/\mu^2$ that is independent of the background fields

$$\mathbf{A} = \begin{bmatrix} \frac{p^2}{\mu^2}(P_m^{(1)} + P_s^{(0)})_{\mu\nu,\alpha\beta} & 0 \\ 0 & \frac{p^2+m^2}{\mu^2} \end{bmatrix} \tag{22}$$

and $\mathbf{B}$ is the background-field-dependent part of $\mathbf{MX}/\mu^2$,

$$\mathbf{B} = \begin{bmatrix} 0 & \frac{1}{4}\frac{p_\mu p_\nu - p^2\delta_{\mu\nu}}{\mu^2}F'(f) \\ \frac{1}{2}F'(f)\frac{p_\alpha p_\beta - p^2\delta_{\alpha\beta}}{\mu^2} & \frac{1}{2}\frac{m^2}{\mu^2}(V''(f)-2) + \frac{1}{2}\frac{\mathbb{K}}{\mu^2}F''(f) \end{bmatrix}. \tag{23}$$

To illustrate how particular finite Green functions are obtained using operator regularization we consider now a restriction of the model to the case $V(f) = F(f) = f^2$. In this restricted model we compute the log dependence of the Green functions $\langle ff\rangle$, $\langle kk\rangle$ and $\langle kff\rangle$. The steps of the calculations are indicated in the appendix, and the results of the calculations are given in (A.7), (A.14), (A.15). Although the restricted model will not yield all the Green function of the full model, it is possible by substitution of $F(f)$ and $V(f)$ in the appropriate paces to generalize the results so that the structure of the Green functions of the full model is apparent. A distinctive feature of operator regularization is that the subtraction of divergences is an implicit part of the regularization, so that all Green functions are automatically finite. The occurrence of the logarithm factor $\ln(m^2/\mu^2)$ in the finite Green function is a reflection of the occurrence of divergences, and consequent subtraction and renormalization, in the usual approaches. For example, the log-dependent part of $\Gamma^{(1)}_{ff}$ is shown in the appendix to be

$$\frac{1}{8\pi}\ln(m^2/\mu^2)\int dx(\tfrac{1}{2}m^2f^2(x)+f(x)f_{,\alpha\alpha}(x)). \tag{24}$$

The log dependence of the $f^2(x)$ term is an indicator of an implicit mass renormalization, while the log dependence of the $f(x)f(x)_{,\alpha\alpha}$ term is an indicator of an implicit linear wavefunction renormalization. In contrast, there is no log dependence whatsoever in $\Gamma^{(1)}_{kk}$, indicating the absence of either a mass or wavefunction renormalization for the spin-2 field. In section 3 we noted that all bare Green functions $\langle f^{2n}\rangle$, and not just $\langle ff\rangle$, are logarithmically divergent. For all $n>1$ these divergences are removed by a nonlinear wavefunction renormalization. Such a nonlinear wavefunction renormalization manifests itself, in operator regularization, by the presence of a dependence on $\ln(m^2/\mu^2)$ in the derivative terms such as $(f(x))^n(f(x))^n_{,\alpha\alpha}$ of the Green function $\langle f^{2n}\rangle$, for $n\geqslant 2$.

To see the occurrence of this feature in the simplest case of $\langle f^4\rangle$ we could either compute directly in the restricted model discussed above the Green function $\langle f^4\rangle$ or we could note the simpler option of using the full model of (14) and replacing $f(x)$ in the second term of (24) above by $\frac{1}{2}F'(f)$. This would generate the log dependence

$$\Gamma^{(1)}_{F'F'}\sim\frac{1}{4}\frac{1}{8!}\ln(m^2/\mu^2)\left(\int dx\,F'(f(x))F'(f(x))_{,\alpha\alpha}\right). \tag{25}$$

This generating functional will lead to contributions to $\Gamma^{(1)}_{f^4}$ proportional to terms of the form

$$\ln(m^2/\mu^2)\int dx f^2(x)(f^2(x))_{,\alpha\alpha}. \tag{26}$$

The occurrence of such terms is an indicator of an implicit nonlinear wavefunction renormalization. The log dependence of the $\langle f^4\rangle$ Green function occurs only in the terms with zero and two derivatives. The zero derivative log dependence is an indicator of the implicit renormalization of the function $V(f)$.

5. Concluding remarks

In this paper we have examined a two-dimensional scalar-tensor gauge theory which possesses a number of features that are of some interest. First of all, at the classical level the free Lagrangian for the second-rank symmetric tensor field vanishes so that the dynamics in the quantum theory are determined by the gauge fixing term. Secondly, all the fields in the model are dimensionless. This feature allows the classical Lagrangian to have the non-polynomial interaction terms made necessary by the divergence structure of the quantum theory without prejudicing the renormalizability of the model.

In section 3 we discussed the divergence structure of the bare theory and indicated how all the divergences could be eliminated by means of not only the usual mass and linear wavefunction renormalizations but also a nonlinear wavefunction renormalization and the renormalization of the two non-polynomial functions of the scalar field which enter the classical Lagrangian.

In section 4 we showed how to set up a framework for computing radiative effects in exactly two dimensions without destroying the gauge invariance of the theory. This involves operator regularization, a technique for writing down the regulated (and renormalized) generating functional for one particle irreducible Green functions without encountering any explicit divergences.

In this paper we have restricted our attention to one-loop order. However, operator regularization can also be used to compute Green functions to two-loop order (and beyond). This has been done successfully in the nonlinear sigma model [7] and several scalar field theories [8].

In section 4 and the appendix we considered the finite (renormalized) Green functions $\langle ff \rangle$, $\langle kk \rangle$, $\langle kff \rangle$ and $\langle f^4 \rangle$ and showed how to identify the occurrence of the implicit mass, linear and nonlinear wavefunction and non-polynomial renormalizations by analysing the dependence of these Green functions on $\ln(m^2/\mu^2)$.

The dependence of the regulated generating functional on the radiatively induced mass parameter μ^2 merits consideration. In multiplicatively renormalizable models changes in μ^2 can be compensated for by charges in the renormalized quantities; this is the content of the renormalization group equations. The situation in the model proposed in section 2 is complicated by the nonlinear wavefunction renormalizations and the renormalization of the non-polynomial functions $V(\phi)$ and $F(\phi)$. It would be interesting to see if a renormalization group equation can be written down in closed form and to compute the associated renormalization group functions. We hope to return to this question at a later time.

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Appendix

In this appendix we show how to compute finite renormalized Green functions using operator regularization. The calculators are carried out in the restricted model of section 4, i.e. the case where $F(f) = V(f) = f^2$. The matrix $\mathbf{A}$ of (22) is not affected by this restriction, but the matrix $\mathbf{B}$ of (23) is replaced by

$$\mathbf{B} = \begin{bmatrix} 0 & -\frac{1}{2}\frac{p^2}{\mu^2}\theta_{\mu\nu}f \\ -f\theta_{\alpha\beta}\frac{p^2}{\mu^2} & \mathbb{K} \end{bmatrix}. \tag{A.1}$$

Green functions are obtained by using the Schwinger expansion of (21) in the generating functional of (19) and (20) and evaluating the functional trace.

In this appendix we compute the two-point functions $\langle ff\rangle$ and $\langle kk\rangle$ to illustrate the method. Keeping those terms that contribute to the two-point functions, we find that

$$\zeta_{(2)}(s)=\frac{\mu^{2s}}{\Gamma(s)}\int_0^\infty \mathrm{d}t\, t^{s-1}\,\mathrm{Tr}\left\{\frac{(-t)^s}{2}\int_0^1 \mathrm{d}u \exp-\left[(1-u)\begin{bmatrix} p^2(P_m^{(1)}+P_s^{(0)}) & 0 \\ 0 & p^2+m^2\end{bmatrix}t\right]\right.$$
$$\times\begin{bmatrix} 0 & -\frac{1}{2}\lambda p^2\theta f \\ -\lambda f\theta p^2 & 0\end{bmatrix}\exp-\left[u\begin{bmatrix} p^2(P_m^{(1)}+P_s^{(0)}) & 0 \\ 0 & p^2+m^2\end{bmatrix}t\right]$$
$$\left.\times\begin{bmatrix} 0 & -\frac{1}{2}\lambda p^2\theta f \\ -\lambda f\theta p^2 & 0\end{bmatrix}\right\}. \tag{A.2}$$

As in two dimensions $P_m^{(1)}+P_s^{(0)}+P_\omega^{(0)}=1$, we see that

$$\exp[-p^2(P_m^{(1)}+P_s^{(0)})t]=\mathrm{e}^{-p^2t}(P_m^{(1)}+P_s^{(0)})+P_\omega^{(0)}. \tag{A.3}$$

We also note that

$$\omega\theta=0 \tag{A.4}$$

$$\theta(P_m^{(1)}+P_s^{(0)})\theta=1 \tag{A.5}$$

and therefore

$$\zeta_2(s)=\frac{\mu^{2s}}{2\Gamma(s)}\mathrm{Tr}\int_0^\infty \mathrm{d}t\, t^{s-1}\int_0^1 \mathrm{d}u\{\exp[-(1-u)(p^2+m^2)t]fp^4\exp(-up^2t)f$$
$$+\exp[-(1-u)(p^2+m^2)t]\mathbb{K}\exp[-u(p^2+m^2)t]\mathbb{K}\}. \tag{A.6}$$

As the projection operator $P_\omega^{(0)}$ occurring in (A.3) does not contribute to (A.6), the infrared problem occurring in [2] does not arise. We now compute the functional trace in (A.6) by inserting complete sets of states [4, 6]. For $\langle k_{\mu\nu}k_{\lambda\sigma}\rangle$, we work entirely in momentum space so that we obtain the transverse result

$$\zeta_{\mathbb{K}\mathbb{K}}(s)=\frac{\mu^{2s}}{2\Gamma(s)}\int \mathrm{d}p\int\frac{\mathrm{d}q}{(2\pi)^2}\mathbb{K}(p)\mathbb{K}(-p)\int_0^1 \mathrm{d}u\frac{\Gamma(s+2)}{[q^2+u(1-u)(p^2+m^2)]^{s+2}}. \tag{A.7}$$

For $\langle ff\rangle$ it is convenient to use the approach of [8] so that

$$\zeta_{ff}(s)=\frac{\mu^{2s}}{2\Gamma(s)}\int_0^\infty \mathrm{d}t\, t^{s-1}\int_0^1 \mathrm{d}u$$
$$\times\int \mathrm{d}x\,\mathrm{d}y\langle x|\exp[-(1-u)(p^2+m^2)t]f|y\rangle\langle y|p^4\exp(-up^2t)f|x\rangle. \tag{A.8}$$

As

$$\langle x|p\rangle=\frac{\mathrm{e}^{\mathrm{i}p\cdot x}}{2\pi} \tag{A.9}$$

we find that

$$\zeta_{ff}(s)=\frac{\mu^{2s}}{2\Gamma(s)}\int_0^\infty \mathrm{d}t\, t^{s-1}\int_0^1 \mathrm{d}u\int\frac{\mathrm{d}p}{(2\pi)^2}\frac{\mathrm{d}q}{(2\pi)^2}f(x)f(y)$$
$$\times\exp[\mathrm{i}(p-q)(x-y)]q^4\exp\{-[(1-u)(p^2+m^2)+uq^2]t\}. \tag{A.10}$$

If we now shift variables $x \to x+y$ and make the expansion

$$f(x+y)=f(y)+x_\alpha f_{,\alpha}(y)+\frac{1}{2!}x_\alpha x_\beta f_{,\alpha\beta}(y)+\dots \tag{A.11}$$

we can integrate over x using the equation

$$\int \frac{\mathrm{d}x}{(2\pi)^2}\exp[\mathrm{i}(p-q)x]x_{\alpha_1}x_{\alpha_2}\dots=\left[\left(-\frac{\mathrm{i}\partial}{\partial p_{\alpha_1}}\right)\left(-\frac{\mathrm{i}\partial}{\partial p_{\alpha_2}}\right)\dots\right]\delta(p-q). \tag{A.12}$$

Keeping the first three terms in (A.11), and performing the standard integrals that arise, leaves us with the result

$$\zeta_{ff}(s)=\frac{(m^2/\mu^2)^{-s}}{8\pi}\int \mathrm{d}x\left[\frac{-2m^2f^2(x)}{(1-s)(2-s)}+f\ (x)\ f_{,\alpha\alpha}(x)\left(\frac{4}{2-s}-\frac{12}{3-s}\right)\right]. \tag{A.13}$$

This yields the ff contribution to the generating functional

$$\begin{aligned}\Gamma^{(1)}_{ff}&=\tfrac{1}{2}\zeta'_{ff}(0)\\ &=\frac{1}{8\pi}\int \mathrm{d}x\left(\tfrac{1}{2}m^2(f(x))^2\{-\tfrac{3}{4}+L\}+f(x)f_{,\alpha\alpha}(x)\{-\tfrac{1}{6}+L\}\right)\end{aligned} \tag{A.14}$$

where $L\equiv \ln(m^2/\mu^2)$.

We note from (A.7) that $\Gamma^{(1)}_{\mathbb{K}\mathbb{K}}=\frac{1}{2}\zeta'_{\mathbb{K}\mathbb{K}}(0)$ is independent of L. In a similar way we can compute the contributions to $\langle k_{\mu\nu}ff\rangle$ that depend on L, and we find

$$\begin{aligned}\Gamma^{(1)}_{\mathbb{K}ff}&=\tfrac{1}{2}\zeta'_{\mathbb{K}ff}(0)\\ &\sim\frac{1}{16\pi}L\int \mathrm{d}x\,\mathbb{K}(x)f^2(x).\end{aligned} \tag{A.15}$$

This dependence on L indicates an implicit coupling constant renormalization.

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